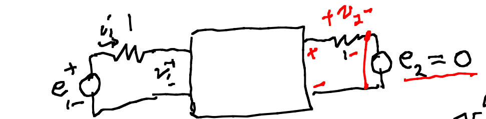
Note Title 9/19/2020

ENEE 610
Tu 10/06/20

Homework 4 new due date = Th 10/08
Normalizations, S21
Homework 5

ideal op-amp & saturating op-amp; exp(At)
Lossless; LPR, LBR

$2v_2^r = v_2 - i_2 = 2v_2$



$e_2 = 0$ if only a load resistor

$e_2=0 \Rightarrow v_2^i = 0 = \frac{1}{2}(v_2+i_2)$

$S_{21} = \left.\frac{v_2^r}{v_1^i}\right|_{v_2^i=0} = \frac{v_2}{e_1/2} = 2\frac{v_2}{e_1}$

$2v_1^i = v_1 + i_1 = e_1$

$$\begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} = 0$$

Normalizations, impedance level & the frequency/time scale

$z(s) \Rightarrow z_{m_R}(s) = \frac{z(s)}{R} \Rightarrow z_L(s) = sL, \quad \frac{z_L(s)}{R} = s\frac{L}{R} = z_{L_{m_R}}(s) \Rightarrow L_{m_R} = L/R$

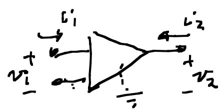
$z_C(s) = \frac{1}{sC} \Rightarrow \frac{z_C}{R} = \frac{1}{RCs}, \; = C_{m_R} = RC, \quad R_1 \rightarrow \frac{R_1}{R} =$ normalized

$z(s) = \frac{N(s)}{D(s)}$; let $s = \omega_n p$, $p =$ normalized frequency

$= z_{m_s}(\omega_n p) = \frac{N(\omega_n p)}{N(\omega_n p)} = \frac{N_{m_s}(p)}{D_{m_s}(p)} \Rightarrow z_C(s) = \frac{1}{sC} = \frac{1}{\omega_n pC} = \frac{1}{pC_n} = z_{C_{m_s}}(p), \; C_n = \omega_n C$

$\Rightarrow$ under R & s scaling $R_n = R_1/R$, $C_n = R\omega_n C$, $L_n = \omega_n L/R$

Ideal op-amp



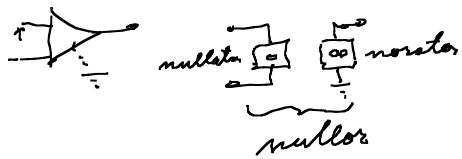
$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} v = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} i$$

$i_1 = 0$

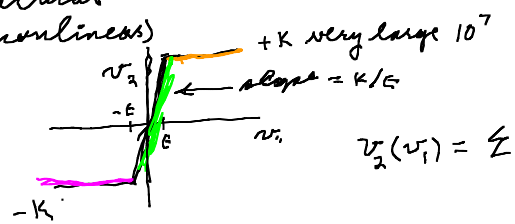
$v_1 = 0$ as a virtual ground (= a tie)

$i_2 =$ is free

$v_2 =$ " " independent of i_2



More Practical (nonlinear)



$$v_2(v_1) = \Sigma \text{ of } \begin{cases} K\,1(v_1 - \epsilon) \\ -K\,1(-\epsilon - v_1) \\ \frac{K}{\epsilon} v_1 \left(1(v_1 - (-\epsilon)) \cdot 1(-v_1 + \epsilon) \right) \end{cases}$$

1 when $-v_1 + \epsilon > 0$, $\epsilon > v_1$

Lossless: mainly L's, C's, gyrators & transformers

Def: $\mathcal{E}(\infty) = 0 = \int_{-\infty}^{\infty} \frac{v^T i + i^T v}{2} dt = \int_{-\infty}^{\infty} \frac{V^{T*}(j\omega) I(j\omega) + I^{T*}(j\omega) V(j\omega)}{2} d\omega = 0$

Energy ↑ lossless

$$= \int_{-\infty}^{\infty} \left[V^{rT*}(j\omega) V^r(j\omega) - V^{iT*}(j\omega) V^i(j\omega) \right] d\omega$$

$$= \int_{-\infty}^{\infty} V^{iT*}(j\omega) \left(S^{T*}(j\omega) S(j\omega) - 1_m \right) V^i(j\omega)\, d\omega$$

$*$ = complex conjugate

$\sqrt{-1} = j \Rightarrow -j$

If rational with real coefficients $S^*(j\omega) = S(-j\omega)$ but $S(s)\big|_{s=j\omega} = S(j\omega) \Rightarrow S(-j\omega) = S(-s)$

$s = \omega/j \Rightarrow s = \sigma + j\omega$

if $\sigma = 0$, $s = j\omega$

By Parseval's theorem

lossless (& passive) $\Rightarrow S^{T*}(j\omega)S(j\omega) = 1_n \Rightarrow$

$= S^T(-j\omega)S(j\omega)$

$\Rightarrow_{\omega = s/j} \Rightarrow \quad S^T(-s)S(s) = 1_n$ if lossless

$\underbrace{S(s) = S^T(-s)}$ no poles on $j\omega$ axis

Ex: C, $Y(s) = Cs$

$S(s) = (1+Y)^{-1}(1-Y) \Rightarrow S(-s) = \frac{1-(-sC)}{1+(-sC)} = \frac{1+sC}{1-sC} = \frac{1}{S(s)}$

$= \frac{1-sC}{1+sC}$ if $C>0$ pole @ $s = -1/C$

For a lossless admittance

$\mathcal{E}(\infty) = 0 \Rightarrow \underbrace{Y(j\omega) + Y^{T*}(j\omega)}_{2} = 0_m \Rightarrow Y(s) + Y^T(-s) = 0_m$, $m = \#$ of ports

Ex: L $Y(s) = \frac{1}{sL}$, a pole $s = 0$, $Y(-s) = \frac{1}{-sL}$, $Y(s) + Y(-s) = \frac{1}{sL} + \frac{1}{-sL} = 0$

pole cancels

for lossless $Y(s) = -Y^T(-s)$ Ex:

Ex: gyrator $\Downarrow$ lossless $Y = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$, $Y^T = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} = -Y$

if passive $Y(s)$ is positive real, if rational it is called PR

if lossless & PR it is called LPR

passive $\Rightarrow$ no poles in $\sigma > 0$ $\qquad Y(s) = -Y^{T}(-s)$ if LPR

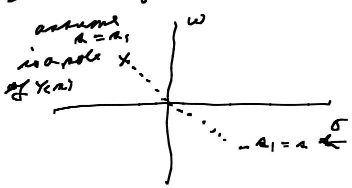
$n=1 \qquad Y(s) = -Y(-s)\,, \qquad Y(s) = \dfrac{N(s)}{D(s)}$ ← polynomial

no zeros of D in $\sigma > 0$

same for $N(s)$ as those zeros are poles of $Z(s)$

the process used is analytic continuation

from $s = j\omega$ to $s = \sigma + j\omega$



$Y(s)$ lossless

this would be a pole of $Y(-s)$ as $-Y(s) = Y(-s)$ can't be

} can't be if LPR $\Rightarrow$ all poles of $Y(s)$ are on the $j\omega$ axis.

$$Y(s) = \frac{n_m s^m + n_{m-1} s^{m-1} + n_1 s + n_0}{d_\delta s^\delta + d_{\delta-1} s^{\delta-1} + \cdots + d_1 s + d_0} = \frac{s^{k_0}(s^2+\sigma_{n_1}^2)^{k_{\sigma n_1}} + \cdots (s^2+\sigma_1^2)^{k_{\sigma 1}}}{s^{k_0}(s^2+\sigma_{k_1}^2)^{q_{\sigma n_1}} \cdots (s^2+\sigma_{k_\delta})^{q}}$$

$n=1$

$=$ only 1st order are allowed due to being PR $= \dfrac{s(s^2+\omega_1^2)\cdots(s^2+\omega_{n_1}^2)}{s^2(s^2+\Omega_1^2)\cdots(s^2+\Omega_{m_1}^2)}$

then make a partial fraction design from it